Contextual Emergence of Mental States from Neurodynamics

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Abstract

The emergence of mental states from neural states by partitioning the neural phase space is analyzed in terms of symbolic dynamics. Well-defined mental states provide contexts inducing a criterion of structural stability for the neurodynamics that can be implemented by particular partitions. This leads to distinguished subshifts of finite type that are either cyclic or irreducible. Cyclic shifts correspond to asymptotically stable fixed points or limit tori whereas irreducible shifts are obtained from generating partitions of mixing hyperbolic systems. These stability criteria are applied to the discussion of neural correlates of consiousness, to the definition of macroscopic neural states, and to aspects of the symbol grounding problem. In particular, it is shown that compatible mental descriptions, topologically equivalent to the neurodynamical description, emerge if the partition of the neural phase space is generating. If this is not the case, mental descriptions are incompatible or complementary. Consequences of this result for an integration or unification of cognitive science or psychology, respectively, will be indicated.

1. Interlevel Relations

Knowledge of well-defined relations among different levels of descriptions of physical and other systems is inevitable if one wants to understand how (elements of) different descriptions depend on each other, give rise to each other, or even imply each other. The most ambitious program in this regard is physical reduction in the sense that higher-level descriptions of features of a system are determined by the description of features at the most fundamental level of physical theory, no matter how remote the higher level is from that most fundamental level. This program assumes that the description of all features which are not included at the fundamental level can be constructed or derived from this level without additional input.

However, already physical examples pose serious difficulties for this program. It has recently been proposed that the concept of *contextual emergence* (Atmanspacher and Bishop, this issue; Bishop and Atmanspacher 2006) addresses such situations more properly. Contextual emergence is characterized by the fact that lower-level descriptions provide necessary, but not sufficient conditions for higher-level descriptions. (Note that such a relation between descriptive levels does not necessarily entail the same relation between ontological levels.) The presence of necessary conditions indicates that lower-level descriptions provide a basis for higher-level descriptions, while the absence of sufficient conditions means that higher-level features are neither logical consequences of lower-level descriptions nor can they be rigorously derived from them alone. Hence, a full-blown reductive program is inapplicable in these cases. Sufficient conditions for a rigorous derivation of higher-level features can be introduced through specifying contexts reflecting the particular kinds of contingency in a given situation.

A key ingredient of this procedure is the definition of some type of *stability condition* (e.g., the KMS condition, due to Kubo, Martin, and Schwinger) based on considerations required to establish the framework of a higher-level description (e.g., thermal equilibrium). This condition is often implemented as a reference state with respect to which an asymptotic expansion is singular in the lower-level state space. Its regularization defines a novel, contextual topology in which novel, emergent features can be rigorously introduced. There is, thus, a mathematically well-defined procedure for deriving higher-level features given the lower level description plus the contingent contextual conditions.

Contextual emergence and the associated identification of appropriate stability conditions may have applications in other domains such as biology and psychology, and, ultimately, in the relationship between the physical and the mental. In this contribution we will address a situation which is particularly difficult because it exceeds the domain of material systems: relations between brain and consciousness. We will discuss the contextual emergence of mental states and related features (psychology, cognitive science) from brain states and related features (neuroscience). Using Harnad's (1990) terms, this refers to the question of how mental symbols and cognitive computation can be grounded in neurodynamics.

More specifically, mental representations will be considered as novel features at the (higher) level of cognition, which have necessary but not sufficient conditions at the (lower) level of neuronal assemblies. In order to identify contexts providing such sufficient conditions, those among the many possible cognitive features that might be relevant or interesting as emergent features must first be identified. Assuming that stability criteria play a role analogous to physical examples, techniques of modeling assemblies in terms of generalized potentials with particular stability properties and corresponding relaxation times or escape times suggest themselves. This can be implemented for powerful modeling tools such as neural networks (Anderson and Rosenfeld 1989) or coupled map lattices (Kaneko and Tsuda 2000).

Other interlevel relations in addition to contextual emergence are strong reduction, radical emergence, and supervenience (cf. Atmanspacher and Bishop, this issue). While we do not think that strong reduction or radical emergence provide clarifying insight for the relation between brain and consciousness, some comments about supervenience are appropriate here. The notion of supervenience characterizes situations in which lower-level descriptions contain sufficient but not necessary conditions for higher-level descriptions.

This scenario has been employed for brain-consciousness relations in the sense that a conscious state with a particular phenomenal content can be multiply realized at the neural level (Kim 1992, 1993). For instance, Chalmers (2000) defines neural correlates of consciousness (NCCs) as neural systems that are correlated with conscious mental states and are minimally sufficient for the occurence of those states.

In this definition, the notion of sufficiency rather than necessity takes into account that different neural states can be correlated with the same conscious state (multiple realization). Our notion of contextual emergence addresses the different question of how neural states are related to conscious states in each individual neural realization. Contextual emergence does not address the distinction between many-to-one and one-to-one relations but tries to elucidate principles which allow us to understand the relationship between mental and neural states itself, even in individual instantiations, in a more profound manner. In this way, supervenience and contextual emergence complement rather than contradict each other. Applying both concepts together may, thus, improve our insight into the nature of mind-brain relations.

2.   Structural Stability in Symbolic Dynamics

The issue of stability plays a prominent role in statistical mechanics. Haag *et al.* (1974) have shown that Gibbs' thermal equilibrium states are uniquely characterized by three stability conditions upon state functionals: (i) stationarity, i.e., expectation values of observables do not change in time; (ii) structural stability, i.e., stability of the dynamics against perturbations; and (iii) "asymptotic abelianness", i.e., temporally distant observables become eventually compatible (see Bratteli and Robinson 1997). From these presuppositions, Haag *et al.* (1974) derived the KMS condition for thermal equilibrium states. In the following we will establish related stability criteria for symbolic dynamics.

Consider a classical time-discrete, invertible dynamical system (X, Φ) given by a compact Hausdorff space as its phase space X and a map $\Phi : X \to X$. The flow of the system is generated by the time iterates $\Phi^t, t \in \mathbb{Z}$, i.e., $t \mapsto \Phi^t$ is a one-parameter group for the dynamics. Then, the function space of complex-valued continuous functions over X, $\mathfrak{A} = C(X)$, yields a C*-algebra of classical observables for that dynamical system.

The states of such a C*-dynamical system are linear, positive, normalized functionals $\rho : \mathfrak{A} \to \mathbb{C}$. For classical dynamical systems they correspond to probability measures μ_ρ over the phase space X, such that $\rho(f) = \int_X f(x) \, d\mu_\rho(x)$ for $f \in C(X)$. While pure states can be identified with single points in phase space $x \in X$, non-pure states are statistical states given by measures μ_ρ that are not concentrated on a single point.

In its simplest sense, the stability of a dynamical system refers to the stability of a point $x^* \in X$ under the flow Φ^t: $x^* = \Phi(x^*)$, i.e., x^* is a fixed-point attractor. Limit cycles or higher-order tori as attractors can be related to fixed points by the technique of Poincaré sections. In general, attractors are invariant sets $A \subset X$, such that $\Phi(A) \subseteq A$ and $\Phi^{-1}(A) \subseteq A$. This invariance property of A extends to probability measures μ according to $\mu(\Phi^{-1}(A)) = \mu(A)$ which are called *stationary* or *invariant measures*. Likewise, a statistical state ρ_μ over the algebra of continuous functions assigned to the measure μ has the invariance property. The invariance of thermal equilibrium states is the first postulate by Haag *et al.* (1974).

Structural stability refers to perturbations in the function space of the flow map Φ. The system (X, Φ) is called *structurally stable* if there is a neighborhood $\mathcal{N}$ of Φ such that all $\Psi \in \mathcal{N}$ are topologically equivalent with Φ.[1] As Haag *et al.* (1974) pointed out, the concept of structural stability is closely related to that of ergodicity. An invariant probability measure μ is said to be *ergodic* under the flow Φ if an invariant set A, has either measure zero or one: $\mu(A) \in \{0, 1\}$. If μ is non-ergodic, there is an invariant set A with $0 < \mu(A) < 1$ corresponding to an accidental degeneracy. Such degeneracies are not stable under small perturbations. Hence, non-ergodic systems are in general not structurally stable (Haag *et al.* 1974).

Thermal equilibrium states are given by invariant, ergodic measures over X. Beyond fixed points and limit tori, more complicated attractors are mixing in addition. Mixing refers to the loss of temporal correlations among the observables of a dynamical system. Formally, a measure μ is called *mixing* if $|\mu(A \cap \Phi^{-t}(B)) - \mu(A)\mu(B)| \overset{t \to \infty}{\longrightarrow} 0$ for all measurable sets A, B (Luzzatto 2006). This property can be rephrased by the correlation of observables $f, g \in \mathfrak{A}$ at time t: $C_t(f, g) = |\rho_\mu(f \cdot g \circ \Phi^t) - \rho_\mu(f) \cdot \rho_\mu(g)|$ where ρ_μ is the statistical state assigned to the measure μ. If $C_t(\chi_A, \chi_B) \overset{t \to \infty}{\longrightarrow} 0$ for characteristic functions χ_A, χ_B of the sets $A, B \subset X$, μ is mixing (Luzzatto 2006). Interestingly, Haag *et al.* (1974) derived this loss of correlations from a more fundamental, purely algebraic stability property called "asymptotic abelianness" (Bratteli and Robinson 1997). The mixing property of a state ρ follows from the asymptotic abelianness of the algebra under the assumption that ρ is *relatively pure*, i.e., ρ cannot be decomposed into a convex sum of invariant states (ρ might be decomposable into non-invariant states, however). Relatively pure states have sharp expectation values and correspond, therefore, to thermodynamic macrostates (Shalizi and Moore preprint).

Stationarity (invariance), structural stability (ergodicity) and asymptotic abelianness (mixing) are important for the investigation of nonlinear dynamical systems. Many rigorous results are known for *hyperbolic systems* where either the whole phase space possesses a hyperbolic structure (Anosov diffeomorphisms) or there is a hyperbolic attractor. Anosov diffeomorphisms are known to be structurally stable (Robinson 1999), and systems with hyperbolic attractors have invariant, ergodic and mixing probability measures due to a theorem by Sinai, Ruelle and Bowen (Ruelle 1968, 1989). For non-hyperbolic systems, much less is known (cf. Viana *et al.* 2003).

Now let us introduce the notion of *epistemic* observables. For this purpose, consider a piecewise constant function f over the phase space X. Such a function is generally not overall continuous and does therefore not belong to the C*-algebra $\mathfrak{A} = C(X)$ of observables. Instead, it belongs to the larger W*-algebra[2] of $\hat{\mu}$-essentially bounded epistemic observables $L^\infty(X, \hat{\mu})$ that are contextually defined by a reference probability measure $\hat{\mu}$ on

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[1]Two maps Φ, Ψ are called *topologically equivalent*, or *conjugated*, if there is a homeomorphism h such that $h \circ \Phi = \Psi \circ h$.

[2]The relationship between C*- and W*-algebras can be illustrated in the following way. Regarding a C*-algebra $\mathfrak{A}$ as a complex vector space one can construct the dual $\mathfrak{A}^*$ of linear functionals containing the states over $\mathfrak{A}$. This is again a vector space that becomes a Hilbert space in the GNS construction and has a dual $\mathfrak{A}^{**}$. The original C*-algebra $\mathfrak{A}$ can be canonically embedded in $\mathfrak{A}^{**}$ by $a(\rho) = \rho(a)$ where $\rho \in \mathfrak{A}^*$, the right-hand side $a \in \mathfrak{A}$, and the left-hand side $a \in \mathfrak{A}^{**}$. Hence, $\mathfrak{A}^{**}$ inherits the properties of $\mathfrak{A}$ (including the C*-property). The fact that it has a Hilbert space as its predual turns it into a W*-algebra. The bidual $\mathfrak{A}^{**}$ is generally much larger than $\mathfrak{A}$ and contains the epistemic observables.

the phase space X used for a Gel'fand-Naimark-Segal (GNS) construction (Primas 1998, Atmanspacher and Bishop, this issue). Two states $x, y \in X$ are called *epistemically equivalent with respect to f* if $f(x) = f(y)$ (beim Graben and Atmanspacher 2006). Epistemically equivalent states are not distinguishable by means of the observable f. The classes of epistemically equivalent states partition the phase space X into disjoint sets A_i.
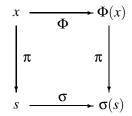
A finite partition of X, $\mathcal{P} = \{A_i | i \leq I\}$, $A_i \cap A_j = \emptyset$ ($i \neq j$), $\bigcup_i A_i = X$, also called a *coarse-graining*, yields a symbolic dynamics (Lind and Marcus 1995) of the system (X, Φ) in the following way: Taking the finite index set of the partition as an alphabet $\mathbf{A}$ of cardinality I, one assigns to each initial condition $x_0 \in X$ a bi-infinite sequence $s = \ldots a_{i_{-1}} a_{i_0} . a_{i_1} a_{i_2} \ldots$ of symbols $a_{i_k} \in \mathbf{A}$ according to the rule $x_0 \mapsto s$, if $\Phi^t(x_0) \in A_{i_t}$, $t \in \mathbb{Z}$ (the dot indicates the origin of the time scale). This mapping $s = \pi(x_0)$ is continuous in the topology of the space of sequences $\Sigma = \mathbf{A}^{\mathbb{Z}}$. Accordingly, the first iterate $x_1 = \Phi(x_0)$ of x_0 is mapped onto the sequence $s' = \ldots a_{i_{-1}} a_{i_0} a_{i_1} . a_{i_2} \ldots$. Therefore, the sequence s' is obtained by shifting all symbols of s one place to the left.

A symbolic dynamical system is given by (Σ, σ) where $\sigma(s) = s'$ is the left-shift. Since the dynamics on Σ is trivially represented by the shift σ, all important information is now encoded in the structure of the symbolic sequences s. Therefore, symbolic dynamics deals with syntax and pattern analysis (Lind and Marcus 1995, Keller and Wittfeld 2004, Steuer *et al.* 2004, Steuer *et al.* 2001).

The systems (X, Φ) and (Σ, σ) are related to each other by

$$\pi \circ \Phi = \sigma \circ \pi, \tag{1}$$

which can be represented diagrammatically as:

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\begin{array}{ccc}
x & \xrightarrow{\ \ \Phi\ \ } & \Phi(x) \\
\downarrow{\scriptstyle \pi} & & \downarrow{\scriptstyle \pi} \\
s & \xrightarrow{\ \ \sigma\ \ } & \sigma(s)
\end{array}
$$

where $\pi : X \to \Sigma$ acts as an *intertwiner*. If π is continuous and invertible and its inverse π^{-1} is also continuous, the maps Φ and σ are topologically equivalent. In this case the partition $\mathcal{P}$ is called *generating*. For *generating partitions*, the correspondence between the phase space and the symbolic representation is one-to-one: each point in phase space is uniquely represented by a bi-infinite symbolic sequence and *vice versa*. Additionally, all topological information is preserved.

Generating partitions are generally hard to find. However, it is known that hyperbolic systems possess generating partitions for which the resulting symbolic dynamics is a Markov chain (Sinai 1968a,b, Bowen 1970). The partitions that achieve this are so-called Markov partitions. The symbolic dynamics obtained from a Markov partition is a *shift of finite type*. This can be seen by defining an $I \times I$ stochastic matrix $P_{ij} = \mu(\Phi^{-1}(A_i) \cap A_j)/\mu(A_i)$ (Froyland 2001), where μ is a probability measure. The associated transition matrix $T_{ij} = \text{sgn}(P_{ij})$ provides a subset $\Sigma_T \subset \Sigma$ of admissible sequences. The sequence $s = \ldots a_{i_{-1}} a_{i_0} . a_{i_1} a_{i_2} \ldots$ belongs to Σ_T if $T_{a_{i_k} a_{i_{k+1}}} = 1$ (i.e., the transition from

a_{i_k} to $a_{i_{k+1}}$ is allowed). The left-shift restricted to Σ_T yields then a subshift of finite type $(\Sigma_T, \sigma|_{\Sigma_T})$.

Assume that a coarse-grained description of a dynamical system (X, Φ) is such a shift of finite type $(\Sigma_T, \sigma|_{\Sigma_T})$. Then we can distinguish two important cases. In the first case, either the matrix T itself or some power T^l ($l > 1$) of T is diagonal. If T is diagonal, the cells A_i of the partition $\mathcal{P}$ are invariant sets under the flow Φ. That is, the partition is coarse enough to capture the asymptotically stable fixed points and limit tori together with their basins of attraction of a multistable dynamical system. Such systems are structurally stable unless they give rise to bifurcations. If the l-th power of T is diagonal, the admissible sequences of the symbolic dynamics are periodic and T is called *cyclic*. This means that the boundaries of the partition are transversally intersected by a limit torus, which is asymptotically stable as well. The space of symbolic sequences Σ_T for these systems can be equipped with invariant, ergodic measures by taking Dirac measures for the periodic sequences.

The second important case refers to an *irreducible* transition matrix T, i.e. there is a number l such that T^l is positive. Then the corresponding shift of finite type $(\Sigma_T, \sigma|_{\Sigma_T})$ is an ergodic and mixing Markov chain where the eigenvector p^* to eigenvalue one of the stochastic matrix P corresponds to a unique invariant, ergodic measure that is mixing in addition to the first case above (Ruelle 1968, 1989). A well-elaborated theory relates these measures to KMS states in algebraic quantum statistics (Olesen and Petersen 1978, Bratteli and Robinson 1997, Pinzari *et al.* 2000, Exel 2004), at least for structurally stable hyperbolic systems. Such systems have Markov partitions enabling the construction of thermal equilibrium KMS states (under certain conditions) which are also structurally stable (Robinson 1999). Furthermore, Markov partitions are generating and, thereby, admit a symbolic dynamics that is topologically equivalent to the underlying phase space dynamics.

To conclude, subshifts of finite type $(\Sigma_T, \sigma|_{\Sigma_T})$, characterized by an $I \times I$ transition matrix T, are structurally stable if T is either cyclic (i.e. there is an $l \geq 1$ such that T^l is diagonal) or if T is irreducible. In both cases, the existence of invariant and ergodic measures ensure stability conditions as required for the contextual emergence of epistemic observables and associated states in a partitioned phase space.

3. Contextual Emergence of Mental States from Neural States

Let us now consider a neurodynamical system $N = (X, \Phi)$ with phase space X described by neural observables $f_i : X \to \mathbb{R}$ (e.g. spike rates or action potentials or somato-dendritic membrane potentials of neurons) such that $x \in X$ is a point or, likewise, an *activation vector* of a neural population given by the values $(f_i(x))_{i \leq n} \in \mathbb{R}^n$ for n degrees of freedom. In the following subsections we address three different ways of introducing epistemic observables on such a phase space. The structural stability of their associated symbolic dynamics, which is of key significance for contextual emergence, will be emphasized in particular.

3.1. Neural Correlates of Consciousness

There is a great variety of conscious mental states forming a *mental state space Y*. Mental states range from coarsest-grained ("just being conscious") to finer-grained states such as

wakefulness versus sleep, dreaming, hypnosis, attentiveness, etc.[3] Even more refined are states of consciousness associated with specific *phenomenal content* (Chalmers 2000).

It is generally assumed that some neural system N with phase space X is correlated with particular mental states $C \in Y$. They can be related to epistemic observables $p : X \to \{0, 1\}$, where $p(x) = 1$ if the activation vector x is actually correlated with the mental state C. A *phenomenal family* $\mathcal{P} = \{C_1, \ldots C_I\}$ is a Boolean classification of pairwise disjoint states that cover the whole mental state space Y (Chalmers 2000). In other words, $\mathcal{P}$ provides a partition of the mental state space Y into I states C_i. The whole mental state space can then be represented by a system of such partitions of different coarse grainings: At the lowest level there is a binary partition defining mental states of "being conscious" and "not being conscious". At subsequent levels, there are more refined partitions defining, for instance, states of "wakefulness", "sleep", and altered states (e.g. hypnosis), again covering the entire mental state space Y.

According to Chalmers (2000), a neural correlate of consciousness (NCC) can be characterized by a minimal sufficient neural subsystem N that is correlated with a conscious state $C \in Y$. This characterization refers to the interlevel relation of supervenience. The sufficiency of N means that the activity of N implies being in conscious state C.

From the point of view of this contribution, however, it is also appropriate to look for necessary conditions for a neural subsystem N whose activation is correlated with the conscious state C in the sense of contextual emergence. Being in a conscious state C implies then the activity of N, so that this activity is a necessary condition for C.

Suppose that N is an NCC for a conscious state $C_i \in \mathcal{P}$ with multiple realizations by different activation patterns of N. Then different neural states, $x, y \in X$, are sufficient for the conscious state C_i. Since $p_i(x) = p_i(y)$, x and y are epistemically indistinguishable from one another and, hence, epistemically equivalent with respect to the observable p_i corresponding to the mental state $C_i \in \mathcal{P}$. In this sense, the partition $\mathcal{P}$ of the mental state space Y induces a partition $Q = \{A_1, \ldots A_I\}$ of the neural state space X into classes of epistemically equivalent neural states. Labeling the cells A_i of Q by symbols a_i of a finite alphabet $\mathbf{A}$, we obtain a symbolic representation of the mental states, emerging from the neural state space by the mapping $\pi : \mathcal{P} \to \mathbf{A}$, $\pi(C_i) = a_i$. The dynamics of states a_i in $\mathbf{A}$ is a discrete sequence of symbols as a function of time, establishing a symbolic dynamics. If the transitions between states of consciousness can be described by an $I \times I$ transition matrix T, the mental symbolic dynamics is of finite type.

A coarse-grained partition of X implies neighborhood relations between states in Y that are different from those in the underlying neural phase space X; in this sense it implies a change of topology. Also, the algebra of mental observables differs from that of neural observables. Obviously, these two differences depend essentially on the choice of the contextual partition of Y, based on the choice of a phenomenal family, inducing the partition of X. We will now show that a particular concept of stability is crucial for a proper choice of such a partition and, thus, crucial for a properly conceived relation between X and Y.

The crucial demand for contextual emergence is that the equivalence classes of neural

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[3]A recent empirically based study concerning the relation between neural and mental state space representations for wakefulness versus sleep and other, subtler examples (selective attention, intrinsic perceptual selection) is due to Fell (2004). For alternative state space approaches see Wackermann (1999) and Hobson *et al.* (2000), and the following subsection.

states in X and, hence, the mental states in Y be structurally stable (in the sense of Sec. 2) under the dynamics in X. Consider, e.g., the partition of Y into the mental states "wakefulness" and "sleep" leading to two disjoint sets in X. Given an appropriate discretization of time, the transition matrix T is cyclic with $T^2 = E$ (E denoting the 2×2 unit matrix). That is, the coarse-grained description provides a limit torus. By contrast, a sufficiently fine-grained partition of Y into mental states of different phenomenal content would have to be described by a high-dimensional irreducible transition matrix T since any such state should be connected to any other state by a symbolic trajectory of sufficient length. In this case the resulting symbolic dynamics is an ergodic, mixing Markov chain with a distinguished KMS equilibrium state (Pinzari *et al.* 2000, Exel 2004).

These stationary and structurally stable symbolic dynamical systems have strikingly different consequences (beim Graben and Atmanspacher 2006). While fixed points and limit tori do not allow for generating partitions (beim Graben 2004), aperiodic Markov chains can be obtained from Markov partitions which are generating. Generating partitions admit a continuous approximation of individual points in the neural phase space X by symbolic sequences in **A** with arbitrary precision. Hence, the neural description in X and the coarse-grained, mental description in Y are topologically equivalent.

This shows that the generating property of a partition is an important constraint for a viable symbolic description of a system. Although this is a clear-cut criterion, generating partitions are notoriously difficult to find in practice, and they are explicitly known for only a few examples. Nevertheless, they are viable candidates for the implementation of a stability criterion appropriate for the contextual emergence of mental states. A related stability constraint has been proposed recently (Werning and Maye 2004, this issue). An alternative approach, focusing on information constraints rather than stability, is due to Shalizi and Moore (preprint).

3.2. Macroscopic Neural States

Another approach, leading to coarse-grained neural states without involving mental states is based on mass potentials such as local field potentials (LFP) at the mesoscopic and the electroencephalogram (EEG) at the macroscopic level of brain organization. Let $F : X \rightarrow \mathbb{R}$ be such an observable given by a mean field

$$F(x) = \sum_i f_i(x) \,, \tag{2}$$

where the sum extends over a population of n neurons and f_i denotes a projector of X onto the i-th coordinate axis measuring the microscopic activation of the i-th neuron.

Similar to the previous subsection, the outcomes of F have multiple realizations since the terms in the sum in Eq. (2) can be arranged arbitrarily. Therefore, two neural activation vectors x, y can lead to the same value $F(x) = F(y)$ (e.g., when $f_i(x) - \varepsilon = f_j(x) + \varepsilon, i \neq j$), so that they are indistinguishable by means of F and, therefore, epistemically equivalent. If the equivalence classes of F in X form a finite partition $Q = \{A_1, \ldots A_I\}$ of X, we can again assign symbols a_i from an alphabet **A** to the cells A_i and obtain a symbolic dynamics. In this way, experimentally well-defined meso- and macroscopic brain observables, LFP and EEG, form a coarse-grained description of the underlying microscopic neurodynamics.

It should be emphasized that related approaches do not involve any reference to concrete mental or conscious states. Whether or not one wants to relate corresponding coarse-grained neural states to mental states is left open (for attempts in this direction, see Fell (2004), Wackermann (1999) and Hobson *et al.* (2000)).

Coarse-grainings based on the symbolic encoding of EEG time series became increasingly popular in recent years (Keller and Wittfeld 2004, beim Graben *et al.* 2000, Frisch *et al.* 2004, Frisch and beim Graben 2005, Drenhaus *et al.* 2006, Schack 2004, Steuer *et al.* 2004). Since such partitions are not induced by well-defined mental observables, it is unclear whether the stability conditions required for contextual emergence are satisfied. It is, thus, particularly important to check this carefully.

One option to do this is to look for Markov partitions of the phase space which minimize correlations between their cells, thus creating a Markov process for the symbolic dynamics of the meso- or macro- observables if the dynamics in X is chaotic.[4] Since Markov partitions are generating, they can be operationally identified by the fact that the dynamical entropy for a generating partition is the supremum over all possible partitions, the so-called Kolmogorov-Sinai entropy (see Atmanspacher (1997) for an annotated introduction). Iterative partitioning algorithms in this and similar contexts have been discussed by Froyland (2001): Starting with an initial partition, those sets which contribute to the greatest mass of the assumed invariant ergodic measure are refined iteratively. Optimal partitions are thereby generated by a dynamics in "partition space".

Alternatively, the measured meso- or macroscopic observables can be analyzed by segmentation techniques (Lehmann *et al.* 1987, Wackermann *et al.* 1993, Hobson *et al.* 2000, Hutt 2004, Schack 2004).[5] A recent proposal to implement this is due to Froyland (2005): One tries to partition the space X into *almost invariant sets* such that trajectories spend most of the time within individual cells of the partition, and transitions between cells are likely at larger time scales. In this way, the dynamics on short time scales is described by cyclic transition matrices, whereas large time scales yield descriptions by Markov processes with irreducible transition matrices. The separation of time scales provides, then, a contextual criterion for properly defined macroscopic brain states. For a related approach see Gaveau and Schulman (2005).

3.3. Remarks on Symbol Grounding

The *symbol grounding problem* posed by Harnad (1990) refers to the problem of assigning meaning to symbols on purely syntactic grounds, as proposed by cognitivists such as Fodor and Pylyshin (1988). This entails the question of how conscious mental states with phenomenal content can be characterized by their NCC. Chalmers (2000) defined an NCC for phenomenal content as a neural system N with "systematicity in the correlation", meaning that the representation of a content in N is correlated with a representation of that content in consciousness. In other words, there should be a mapping from the neural state space X onto the space of conscious states Y such that regions in X are related to phenomenal

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[4]Evidence for chaotic brain processes has often been reported (cf. Kaneko and Tsuda 2000, and references therein).

[5]Lehmann *et al.* (1987) called the corresponding states "brain microstates" or "atoms of thought", expressing the suggestion that they correspond to elementary "chunks" of consciousness.

contents in Y. This mapping differs from the mapping required for contextual emergence as discussed in Sec. 3.1. For a neural representation of content further constraints are crucial.

First, all states in $B \subset X$ representing the same content C should be similar in some respect: there should be a mapping $g : X \to X$, let us call it a *gauge transformation*, such that B is invariant under g, $g(B) \subset B$. In this sense, g is a *similarity transformation*. On the other hand, graded differences in phenomenal similarity should be reflected by topologically neighboring regions in phase space. One would, therefore, require the mapping g to be a homeomorphism, leading to *topographic mappings* of contents (Chalmers 2000).

A second requirement is the *compositionality* of representations (Fodor and Pylyshin 1988, Werning and Maye 2004). Compositionality refers to the relation between syntax and semantics insofar as the meaning of a composed (or "complex") symbol is a function of the meanings of its constituent symbols and the way they are put together. A prerequisite for compositionality is the existence of syntactic rules determining which composites are *constituents* of a language and which are not. (In our approach, constituents are admissible (sub-)sequences in the corresponding symbolic dynamics (beim Graben 2004).)

According to Harnad, these constraints need to be combined with his proposal that symbols must be grounded in embodied cognition. They represent objects or facts from the environments of physically embodied agents that collect information by their sensory apparatuses and act by their motor effectors. While Harnad (1990) suggests a hybrid architecture consisting of a neural network as an invariance detector and a classical symbol processor to meet the compositionality constraint, we shall discuss the alternative of a unified neurodynamical system.

This can be achieved using the notion of *conceptual spaces* as discussed by Gärdenfors (2004). A conceptual space is a vector space spanned by quantitative observables. The conceptual space for color, e.g., can be constructed as the three-dimensional RGB coordinate system or an equivalent representation supplied by the cones in the retina (Steels and Belpaeme 2005). According to Gärdenfors (2004) a *natural concept* is then a convex region in a conceptual space such that all elements in that region are similar in a particular context. Implementing conceptual spaces by neural systems, we arrive again at partitions of neurodynamical phase spaces. The idea of gauge invariance yields partitions of finest grain, corresponding to "natural kinds" (Carnap 1928/2003, Quine 1969), that might be too refined for other contexts.

Such contexts can be supplied by pragmatic accounts. Suppose a toy-world in which only orange objects are eatable, and all other objects are not (Steels and Belpaeme 2005). Then, a binary partition of color space into "orange" and "non-orange" will be sufficient for an agent to survive. Thus, survival (or successful communication) serve as contextual constraints for the emergence of cognitive symbols. Symbol grounding corresponds then to *categorization* of conceptual spaces driven by pragmatic goals.

The contextual emergence of symbols in partitioned conceptual spaces raises the question of the stability of the symbols. The dynamics that has to be taken into account now is, however, not neurodynamics but rather sociodynamics: the evolution of populations of cognitive agents. (Neurodynamically, concepts are static objects given by the cells of a partition.) An interesting approach in this sense has been developed within the framework of *evolutionary game theory* (Steels and Belpaeme 2005, Jäger 2004, van Rooy 2004). In these models the phase space is spanned by the population numbers of agents with com-

peting strategies. The outcome of the games is assessed by a utility function which in turn determines the number of offspring of the players. In cognitive applications of evolutionary game theory, offspring means adoption of the winning strategy by other players.

If categories or concepts are given by partitions of conceptual spaces, competing strategies are different partitions of the same local conceptual spaces shared by different agents. Evolutionary game theory then describes a dynamics in partition space similar to the search for optimal partitions by iterative algorithms (Froyland 2001). Evolutionary stable strategies are asymptotically stable fixed points in evolutionary game theory (Jäger 2004). This stability criterion means that cultural evolution grounds symbols in shared partitions of local conceptual spaces of cognitive agents.

The structural stability of dynamically evolving partitions can be illustrated by the "naming game" (van Rooy 2004). When a categorization in conceptual space is fixed the cells are labeled by symbols of an alphabet **A**. This can be done arbitrarily by convention, or it can be achieved by another pragmatic game that optimizes the utility reward. For instance, assume that two meanings m_1, m_2 assign two symbolic forms f_1, f_2, that m_1 is less complex than m_2, and the same for the forms f_1, f_2. For such a scenario, van Rooy (2004) found only two evolutionarily stable strategies: the *Horn strategy* which assigns more complex forms to more complex meanings, and the *anti-Horn strategy* performing otherwise. Since the basin of attraction of the Horn strategy is larger than that of the anti-Horn strategy (Jäger 2004), the Horn strategy provides a higher degree of structural stability.

4.   Compatibility of Psychological Descriptions

It is an old and much discussed question whether and, if yes, how psychology could become a unified science, integrating the many approaches and models that constitute its contemporary situation. It is often argued that the largely fragmented appearance of psychology (and cognitive science as well) is due to the fact that psychology is still in a preparadigmatic, "immature" state. Some have even argued that this situation is unavoidable (e.g., Koch 1983, Gardner 1992) and should be considered as the strength of psychology (e.g., Viney 1989, McNally 1992) rather than an undesirable affair.

From the perspective of the philosophy of mind, arguments against the possibility of a unified science of psychology have been presented as well. Most prominent are the accounts of Kim (1992, 1993) and Fodor (1997), both using the scheme of multiple realization in the framework of supervenience to reject unification. Shapiro (in press) has recently pointed out particular weak points in their arguments.

On the other hand, there is a growing interest in articulating visions for a unified science of psychology, cognition, or consciousness (see, e.g., Newell 1990, Anderson 1996). Recently, various approaches have been proposed to reach a degree of coherence comparable to established sciences as, e.g., physics with well-defined relations between its different disciplines. Examples are approaches such as "psychological behaviorism" (Staats 1996, 1999), "unified psychology" (Sternberg and Grigorenko 2001, Sternberg *et al.* 2001), and the "tree of knowledge system" (Henriques 2003). A key feature in the latter program is the *commensurability* of competing approaches in psychology, explicated by Yanchar and Slife (1997) and Slife (2000).

This section presents a way in which the notion of commensurable models can be implemented in a formally rigorous fashion. A suitable way to formulate commensurability in technical terms is related to the concept of *compatibility*. Briefly speaking, two models are considered as commensurable if they are compatible in the sense that there exist well-defined mappings between them. If this is not the case, they are incompatible. It turns out that the scheme of contextual emergence provides some detailed and clarifying insights how to proceed in this regard. The two levels of description whose interlevel relations are significant for this purpose are those of neurobiology and psychology or cognitive science, respectively. Compatible and incompatible implementations of cognitive symbol systems have recently been discussed by beim Graben (2004).

A key result of the work by beim Graben and Atmanspacher (2006) is that a non-generating partition is incompatible with any other partition (even if this is generating) in the sense that there is no well-defined mapping between the partitions.[6] As a consequence, models based on such partitions are incompatible as well. Since any *ad hoc* chosen partition is quite unlikely to be generating, it may be suspected that the resulting incompatibility of models based on such partitions is the rule rather than the exception. While incompatibility may admit the possibility of "partially coherent" models, the case of maximal incompatibility, also called complementarity, excludes any coherence between different models completely.

At this point it should be clear that our notion of incompatibility is more subtle than a "logical incompatibility" (Slife 2000) in the sense that two models are simply negations of each other. Also, it would be interesting to compare Slife's (2000) complementary models with our formal approach in terms of maximally incompatible models, which are basically incoherent only in a Boolean framework. From a perspective admitting non-Boolean descriptions, the notion of coherence acquires a more comprehensive meaning, including complementary descriptions as representations of an underlying, more general description (see Primas 1977).

With these remarks in mind, incompatible models due to non-generating partitions represent a significant limit to the vision of a unified or integrative science of psychology. Or, turned positively, such a unification will be strongly facilitated if the approaches to be unified are based on generating, hence compatible, partitions that are structurally stable and induced by well-defined mental or cognitive states. As mentioned, it is a tedious task to identify such generating partitions. Nevertheless, the necessary formal and numerical tools are available today and can be implemented by the symbolic description using shifts of finite type and transition matrices. All one has to do is find transitions between mental states that are irreducible, yielding a stationary, ergodic, and mixing Markov chain with distinguished KMS states.

If there is a good deal of empirical plausibility for a particular partition, one might hope that this implies that such a partition is generating (at least in an approximate sense) and, thus, that the corresponding mental or cognitive states are stable (in the sense of the KMS condition). However, there may be cases of conflict between the empirical and the theoretical constraint on a proper partition. In such cases, one has to face the possibility that the

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[6]Two partitions $\mathcal{P}_1$ and $\mathcal{P}_2$ are (in)compatible if their σ-algebras are (not) identical up to μ-measure zero. Two partitions are maximally incompatible, or complementary, if their σ-algebras are disjoint up to the entire phase space X (cf. beim Graben and Atmanspacher 2006).

"empirical plausibility" of cognitive states may be unjustified, e.g., based on questionable prejudices. If cognitive states turn out to be dynamically unstable, this theoretical argument against their adequacy is very strong indeed.

Compatible partitions and, consequently, compatible psychological models show another important feature that is occasionally addressed in current literature: the topological equivalence of representations in neurodynamic and mental state spaces (cf. Metzinger 2003, p. 619, and Fell (2004) for empirically based examples). Topological equivalence ensures that the mapping between X and Y is faithful in the sense that the two state space representations yield equivalent information about the system (see Sec. 2). Non-generating, incompatible partitions do not provide representations in Y that are topologically equivalent with the underlying representation in X.

5. Summary

The relation between mental states and neural states is discussed in the framework of a recently proposed scheme of interlevel relations called contextual emergence. According to this proposal, knowledge of the neural description provides necessary but not sufficient conditions for a proper psychological description. Sufficient conditions can be defined by contingent contexts at the cognitive (phenomenal) level and implemented as stability criteria at the underlying neural level.

This procedure has been demonstrated using the terminology of symbolic dynamics at the cognitive level. Equivalence classes of neural states are defined as neural correlates of mental states represented symbolically. Mental states are well-defined if criteria of temporal and structural stability are satisfied for their neural correlates. These criteria can be implemented either by generating or, more specifically, Markov partitions; or by partitions of systems with asymptotically stable fixed points or limit tori. This implies that proper mental or cognitive states must satisfy appropriate stability conditions.

If this is not explicitly taken care of for chaotic systems admitting generating partitions, one has to expect that *ad hoc* selected partitions are not generating. As a consequence, models based on such partitions are incompatible. This may be a possible source of the long-standing problem of how to develop a unified science of psychology. Only for carefully chosen generating partitions it can be guaranteed that different cognitive models are compatible and, hence, can have transparent relations with respect to each other.

Moreover, psychological (or cognitive) models are topologically equivalent with their neurobiological basis only if they are constructed from generating partitions. Without cognitive contexts serving as sufficient conditions for compatibility and topological equivalence, the neurobiological level of description provides only necessary conditions for psychological descriptions.

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